

## Internal auctions for the efficient sourcing of intermediate products

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### Abstract

We examine the problem of setting production levels and selecting an internal supplier for a vital intermediate product. These decisions are being made in a decentralized environment involving internal suppliers that are profit centers with private cost information. Our analysis centers on the use of multi-dimensional auctions for facilitating communication between divisions and for implementing efficiency goals. We demonstrate analytically what the efficiency goals of the purchasing division should be and characterize necessary conditions of the auction process for implementing those efficiency goals.

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### 1. Introduction

Firms in many industries are increasingly using market mechanisms for internal uses such as resource allocation, planning, and investment. Such mechanisms often involve decentralized decision making by divisions in a firm acting as “profit centers” and frequently involve the transfer of parts and services from one division to another. Transfer prices on the exchanged goods and services are created to account for the transfer’s effect on the performance of the divisions involved in the exchange. In this way, transfer prices are used to place an implicit (or in some cases explicit) value on the intermediate products that are traded between divisions of a firm. They also allow for various degrees of decentralization as they can be used to induce division managers to pursue internal markets for

their products, to set production levels, and to conduct trade with outside markets, or any combination of the above.

Transfer prices have been characterized (see (Vaysman, 1991)) as belonging to one of two general categories: cost-based (administered) or negotiated. Cost-based transfer prices are administratively set to reflect the cost of production and therefore allow for efficient production planning and provide a ‘just’ reward to the seller. Classical economic theory dictates that when transfer prices are used to set production levels, they need to be set equal to the marginal cost of supply to ensure efficiency (Hirshleifer, 1956). In the presence of private cost information, the situation becomes more complicated. Transfer prices are then viewed as a mechanism with which to induce truthful revelation of costs (Besanko and Sibley, 1991). Since inducing truthful revelation will typically require paying an “information rent” (see (Baron, 1989)), production levels are optimally set below their economically efficient levels in order to

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recoup some of the information rent from suppliers. This result applies when the purchasing division's goal is to maximize the net profit of its own division. Here, we examine the role of transfer prices in a situation where the objective is to maximize the profit of the firm as a whole. We will show that two prices, a fixed and a marginal (per unit) transfer price, can be used to both induce revelation of supplier's costs and set production at its economically efficient level.

Negotiated transfer prices represent a more decentralized form of decision making. They potentially allow for all key decisions to be made by division managers with little direct oversight by corporate headquarters. There is also a danger of negative outcomes related to the high transaction cost of bargaining and inefficient production levels arising from poorly (or strategically) negotiated prices. As chronicled by Ugarte (1994), the lack of an explicit procedure for allocating resources between decentralized divisions at Hewlett Packard has tended to politicize the decisions and generate a level of distrust with the process. The trade-off between negotiated and costs based procedures has been described as one of flexibility gain versus control loss (Williamson, 1967).

An alternative for setting transfer prices that applies in an internally competitive environment is to let the *internal* market determine the transfer prices. Specifically, this could take the form of a competitive auction between suppliers. Supplier bidding has been tried in several industries, one of the most publicized programs being that of General Motors during the Lopez era (Fleming, 1992). Auctions can be viewed as surrogate models of price formation (Hansen, 1988). From one point of view this practice represents a negotiated solution since a price is being offered by a seller to be accepted by the purchaser. However, optimal auction design can produce what amounts to cost-based transfer pricing by using the mechanism of the auction to induce revelation of costs. Auctions have a further advantage over classic incentive mechanisms in that they require a specific characterization of the private information of suppliers to design the mechanism. An auction is also a more familiar mechanism that would be more practicable to implement than a complicated series of contingent contracts.

The auction process also provides a simple but powerful tool for communicating the preferences of the purchasing division to its potential suppliers. Through the "scoring" of bids, a process we attempt to formalize below, the purchasing division is signalling how it values the relevant attributes of the products that it is purchasing. Suppliers are then able to adapt the design and planning of its products to the preferences of the purchaser. This is an important fact for managers to keep in mind as sometimes these signalled preferences may be unintended, as might have been the case at GM. Some suppliers have alleged that GM's bidding process communicated a preference for low price over quality (Fleming, 1993).

In this paper we analyze an internal market where an assembly division is selecting an internal supplier of an intermediate product. A typical example would be an electronics manufacturer determining an internal source of application specific integrated circuits. Decisions about production levels would ideally depend upon the resulting transfer prices set for the intermediate product. The supplying divisions possess private information about their production costs, which can include fixed costs such as retooling or batch set-up costs. The fixed costs may also reflect the forgone profits of sales outside the company. Knowledge of outside negotiations or expertise in outside markets represents another aspect of the private information possessed by supplying divisions. This is an especially relevant consideration in the semiconductor industry, where several companies that had produced integrated circuits for internal use only are now seeking external markets for these intermediate products (McWilliams, 1993).

Also important is the fact that the cost curves of the potential supplying divisions may cross at some level of production. Thus, the least cost supplier at one level of production may not be the least cost supplier at other production levels. The objective is both to select the most efficient potential supplier and to set the transfer prices in such a way that the correct cost information is transmitted back to the purchaser. Production of both the final and intermediate product can then be set at maximally efficient levels. We characterize an auction mechanism that accomplishes both these goals. Further, under this mechanism it will be a dominant strategy for a

division to trade in this economically efficient manner. In other words, an individual selling division will find it optimal to request an efficient transfer price no matter what strategies it expects the other divisions to adopt. This seems especially relevant in a transfer pricing setting since the introduction of decentralized decision making can lead to distrust between division managers.

A key feature of this auction mechanism is that it calls for two-dimensional bids, ones with both fixed and marginal transfer prices. Multi-dimensional auctions have received much recent attention as a mechanism for implementing allocations of resources with relevant attributes besides price, such as quality (Che, 1993) or operational control (Bushnell and Oren, 1994). Multi-dimensional auctions have also been analyzed in the context of highway construction contracts (Stark, 1974), and the acquisition of mineral rights (Rothkopf and Englebrecht-Wiggans, 1992). As mentioned above, the relative “weights” given to the dimensions of a bid will signal the preferences of the purchaser to the bidders. Here the purpose of the two-dimensional bids is twofold: first, they induce the revelation of costs that can be dependent on more than one dimension of private information and, second, they allow for efficient levels of production by forcing the producing divisions to collect their information rents entirely in the fixed price portion of their bid.

In the next section we outline the structure of the purchasing division’s problem and characterize the optimal solution to that problem. We then translate that problem into the form of an internal market. In Section 3, we formulate the auction mechanism and characterize the strategies and payoffs of the bidders. A function for ‘scoring’ the bids that induces production efficiency as a dominant strategy is developed in Section 4. Section 5 describes the second price auction mechanism which induces both production and selection efficiencies as a dominant strategy equilibrium.

**2. Problem**

We examine procurement decisions that are made in a firm with production divisions that are profit centers. The division managers are therefore inter-

ested in maximizing the profit of their own division. Specifically, we consider the problem of selecting an internal supplier of an intermediate good or service by a purchasing division where the final product is assembled and marketed. We assume that final product  $x$  is produced from intermediate product  $y$  according to the production function  $x = \Phi(y)$  with corresponding assembly or packaging costs  $c(x)$ . Product  $x$  is projected to yield gross revenue according to the continuous function  $R(x)$ . It is assumed that  $\Phi' > 0$ ,  $\Phi'' \leq 0$ ,  $c' > 0$ ,  $c'' > 0$ ,  $R' > 0$  and  $R'' < 0$ .

There are  $n$  possible supplying divisions with corresponding cost functions  $C_i(y)$ ,  $i = 1, \dots, n$ , where  $y$  is the quantity produced. We assume that the cost functions are continuous with possible “jumps” at the origin (to account for set-up or retooling costs). We further assume that the cost functions are the private information of the suppliers. An exclusive supply contract is to be awarded due to either technological constraint, efficiency considerations (e.g., economies of scale), or some qualitative or long term reason such as a desire for uniformity of inputs or the facilitation of learning effects. The “first-best” solution for the firm as a whole is thus to employ supply source  $i$  to produce quantity  $y$ , where  $i$  and  $y$  solve the total profit maximization problem:

$$\begin{aligned} & \text{Max}_{i \in \{1, \dots, n\}} \text{Max}_{y \in \mathbb{R}_+} \Pi(y, i) \\ & = R(\Phi[y]) - c(\Phi[y]) - C_i(y). \end{aligned} \tag{1}$$

The solution to this selection problem invokes two efficiency criteria:

*Production efficiency.* Production efficiency corresponds to the inner maximization and is achieved by setting the input quantity so as to maximize total profit given supplying division  $i$ , i.e.,

$$y_i^* = \text{Arg Max}_y R(\Phi[y]) - c(\Phi[y]) - C_i(y). \tag{2}$$

First order necessary conditions for (2) imply that

$$(R'(\Phi) - c'(\Phi))\Phi'[y_i^*] = C'_i(y_i^*). \tag{2a}$$

*Selection efficiency.* Selection efficiency corresponds to the outer maximization and is achieved by select-

ing the supplier who under production efficiency will attain the maximum net profit, i.e.,

$$i = \text{Arg Max}_i \{R(\Phi[y_i^*]) - c(\Phi[y_i^*]) - C_i(y_i^*)\}. \quad (3)$$

The selection of a supplying division, the quantity of intermediate product to be supplied, and the transfer prices paid will be accomplished with an internal auction in which the quantity to be supplied is a function of the constant *marginal* transfer price  $p$  that is bid. The purchasing division derives an internal demand function for intermediate product  $y$  as a function of marginal transfer price  $p$  from the following optimization problem:

$$D(p) = \text{Arg Max}_y R(\Phi[y]) - c(\Phi[y]) - py.$$

First order conditions of the interior solution characterize this value as the level of  $y$  such that  $(R'(\Phi) - c'(\Phi))\Phi'(y) = p$ . We assume that  $R(\Phi[D(0)]) - c(\Phi[D(0)]) > 0$ , so that the production of  $x$  is profitable when the intermediate good has no cost. There will exist a  $p$  for which the production of  $x$  will no longer be profitable. This value will represent an upper bound on  $p$  which we denote as  $L$ , where  $D(L) = 0$ . The concavity of  $R(\Phi[y]) - c(\Phi[y])$ , along with the assumptions on the boundary solutions, ensures the existence of a non-negative value  $D(p)$ , which is monotone non-increasing in  $p$ .

When the first order characterization of  $D(p)$  is considered, the optimality condition for production efficiency (2a) becomes  $p_i^* = C'_i(D(p_i^*))$ , i.e., marginal transfer price equals marginal cost at the respective level of production. However, for an arbitrary cost function this condition may not necessarily yield a unique price, we therefore will be using the more general optimality condition (2) to characterize production efficiency.

In this paper, we present a first price auction mechanism which, in dominant strategies, implements production efficiency and a generalized second price auction mechanism which achieves both production and selection efficiency in dominant strategies, thus implementing the first-best solution. In both cases, the potential suppliers compete for an exclusive supply contract by submitting a two parameter bid  $\{p, F\}$  where  $p$  specifies the marginal transfer price at which the supplier commits to sup-

ply the good and  $F$  is a lump sum transfer which it requires (or offers as rebate) for undertaking the supply commitment. A publicly known scoring formula  $S(p, F)$  is used to map the two-dimensional bid into a one-dimensional score and the lowest score bidder wins the exclusive supply contract. In the case of a first price auction, the winning bid is paid the lump sum specified in the bid, while in the second price case the paid lump sum is adjusted on the basis of the best losing bid, as will be described later. In both cases, however, the supplier is held to the marginal transfer price specified in the bid.

### 3. Bidding model

As indicated above, we assume that bidders (supplying divisions)  $i = 1, \dots, n$  have privately known cost functions  $C_i(y)$  specifying their total cost of producing  $y$  units of the good to be supplied. The cost functions are continuous except for possible jumps at  $y = 0$  that reflect fixed costs associated with taking on the contract. These costs may include the opportunity costs of foregone commitments to outside markets.

The strategy of each bidder  $i$  consists of specifying a pair of "fixed" and "variable" transfer prices  $\{F_i, p_i\}$ ,  $F_i \in [-K, K]$  and  $p_i \in [0, L]$ , for some sufficiently large values  $K$  and  $L$ , where  $L$  is as defined in Section 2.<sup>1</sup>

The bids are collected in a sealed-bid auction and scored using a publicly known scoring function:  $S(F, p): [-K, K] \times [0, L] \rightarrow \mathbb{R}^1$ . The bid with the lowest score wins the exclusive supply commitment. The quantity transferred is determined by  $D(p)$ , which is known to all parties.

The payoff to the winning bidder  $i$  consists of two parts: sales revenues  $p_i D(p_i)$  from selling the good at his bid price and a fixed payment (which could be positive or negative). The difference between a first price and second price auction will be manifested in that fixed portion of the payoff to the winning bid as described below.

<sup>1</sup> The results described below could also be implemented with an equivalent mechanism in which bidders bid a quantity  $D$  and a total price.

*First price auction.* The fixed portion of the winning bid's payoff is the bid value  $F$ . Thus, the total payoff to bidder  $i$ , if he wins, is

$$\pi_i^I(F_i, p_i) = F_i + p_i D(p_i) - C_i(D(p_i)). \quad (4)$$

*Second price auction.*<sup>2</sup> A natural extension of a Vickrey (1961) second price auction to our two-dimensional setup is to "adjust" the fixed portion of the payoff to the winning bid  $i$  so as to equate its score to the best losing score  $j$ . Thus, under this scheme, the payoff to winning bid  $i$  is

$$\pi_i^{II}(F_i, p_i) = \hat{F}_i + p_i D(p_i) - C_i(D(p_i)),$$

where  $S(\hat{F}_i, p_i) = S(F_j, p_j)$ . (5)

The only remaining degree of freedom available to the purchasing division is the specification of the scoring function. Thus, the remainder of this paper will focus on the characterization of this function and the implication of that characterization.

#### 4. First price auction

Each bidder  $i$  forms a subjective probability of winning the auction as a function of score that arises from that bidder's expectations about its opponents strategies and costs. This probability, which will be denoted as  $P_i(S(F_i, p_i))$ , may depend on the bidders' private information<sup>3</sup> and is assumed to be once differentiable and decreasing in the score. Thus, the expected payoff of bidder  $i$  as a function of its bid is

$$E\{\pi_i^I(F_i, p_i)\} = [F_i + p_i D(p_i) - C_i(D(p_i))] \times P_i(S(F_i, p_i)). \quad (6)$$

<sup>2</sup> This form of second price auction is obviously not unique. It is possible to define alternative forms by adjusting both values of the fixed and variable price in the payoff function so that the score corresponding to the adjusted values equals the best losing score. We choose to adjust only the fixed portion since our objective is to elicit efficient marginal prices in the bids which could be subsequently used by planners to achieve production efficiency.

<sup>3</sup> This is the case when bidder's costs are probabilistically linked. If bidders' cost values are independent of each other, a bidder's subjective probability of winning will depend on that bidder's score, but not on its costs. The results of this paper apply to both cases.

For ease of exposition, we assume, without loss of generality, that all bidders are risk neutral and, hence, will choose their strategy so as to maximize their divisions' expected payoff<sup>4</sup>. Our objective is to characterize a scoring function that will induce each bidder to bid a variable price that equals marginal cost at the corresponding internal demand quantity, a necessary condition for production efficiency of the auction outcome.

**Proposition 1.** *In the first price auction described above, a necessary condition for a Bayes–Nash equilibrium at which the bid variable price equals marginal cost at the corresponding internal demand level is*

$$\frac{\partial S / \partial p}{\partial S / \partial F} = D(p) \quad \text{for all } p \text{ s.t. } p = C'_i(D(p)),$$

$i = 1, \dots, n.$  (7)

**Proof.** First order necessary conditions for maximizing  $E\{\pi_i^I(F_i, p_i)\}$  are

$$\frac{\partial E\{\pi_i^I\}}{\partial p_i} = [p_i D' + D - C'_i(D) D'] P_i + \pi_i^I P'_i \frac{\partial S}{\partial p_i} = 0 \quad (8)$$

and

$$\frac{\partial E\{\pi_i^I\}}{\partial F_i} = P_i + \pi_i^I P'_i \frac{\partial S}{\partial F_i} = 0. \quad (9)$$

Substituting (9) into (8) and setting  $p_i = C'_i(D(p_i))$  yields (7). □

The intuitive interpretation of condition (7) is that the marginal rate of substitution between the fixed and the variable price in the score has to equal the corresponding rate of substitution between the price components in the payoffs made to winning bidders

<sup>4</sup> The assumption of risk neutrality is made to simplify the exposition. However, the results of Propositions 1, 3 and 4 hold for risk averse bidders as well. This can be easily verified by replacing in the proofs to these propositions, expected profits of bidders with expected Von Neumann–Morgenstern utility of bidders.

at the point of truthful bidding of marginal costs. Since the cost functions are private information, the only practical way to select a scoring rule that always meets condition (7) is to require that it be satisfied for all values of  $p$ . This added restriction specifies the scoring rule uniquely up to a monotone transformation (which will not affect the outcome of the auction) as shown below.

**Proposition 2.** *Let  $S(F, p)$  be a strictly monotone increasing differentiable function in both arguments which satisfies the condition*

$$\frac{\partial S/\partial p}{\partial S/\partial F} = D(p) \quad \text{for all } p \in [0, L]$$

and  $F \in [-K, K]$ .

Then  $S(\cdot)$  is of the form

$$S(F, p) = V\left(-F + \int_p^\infty D(p) dp\right), \tag{10}$$

where  $V(\cdot)$  is some strictly decreasing differentiable function.

**Proof.** For any level set of  $S(\cdot)$  we have

$$\left. \frac{dF}{dp} \right|_{S(F,p)=s} = -\frac{\partial S/\partial p}{\partial S/\partial F} = -D(p). \tag{11}$$

Integrating (11) yields

$$F = \int_p^\infty D(p) dp - \Psi(s),$$

where  $\Psi(s)$  represents an integration constant depending on the level  $s$ . The monotonicity condition on  $S(\cdot)$  implies that  $\Psi(s)$  is strictly decreasing. Thus,

$$S(F, p) = s = \Psi^{-1}\left(-F + \int_p^\infty D(p) dp\right),$$

which is equivalent to (10).  $\square$

The argument of the function  $V(\cdot)$  is the purchasing division's surplus resulting from a fixed transfer  $F$  and marginal transfer  $p$  for the good. This value represents the difference between the purchaser's willingness to pay and the bid price for the total

contract. It is analogous to consumer surplus in standard economic theory. We will refer to this quantity as the *purchaser surplus*. Furthermore, the outcome of the auction both for the supplier and the purchaser is invariant to the specific form of the function  $V(\cdot)$ . Hence, without loss of generality we may take  $V(\cdot)$  to be the negative identity so that the score is simply the negative of the purchaser surplus. We show now that the scoring characterized above guarantees production efficiency for the outcome of the first price auction.

**Proposition 3.** *In a two-dimensional first price auction as described above with a scoring formula given by Eq. (10), it is a dominant strategy for each bidder to bid a variable price which maximizes total profit under its cost function.*

**Proof.** Let  $CS_i = CS(F_i, p_i)$  denote the purchaser surplus corresponding to bid  $\{F_i, p_i\}$  i.e.,

$$CS(F, p) = -F + \int_p^\infty D(p) dp. \tag{12}$$

Then, a strategy  $\{F_i, p_i\}$  is equivalent to a strategy  $\{CS_i, p_i\}$  and the expected payoff of bidder  $i$  given in (6) can be expressed as

$$\begin{aligned} E\{\pi_i^1(CS_i, p_i)\} &= \left[ -CS_i + \int_{p_i}^\infty D(p) dp + p_i D(p_i) \right. \\ &\quad \left. - C_i(D(p_i)) \right] P_i(V(CS_i)). \end{aligned} \tag{13}$$

From the definition of  $D(p)$  and the envelope theorem, it follows that

$$\begin{aligned} R(\Phi[D(p)]) - c(\Phi[D(p)]) - pD(p) &= \int_p^\infty D(\rho) d\rho. \end{aligned} \tag{14}$$

Consequently (13) can be rewritten as

$$\begin{aligned} E\{\pi_i^1(CS_i, p_i)\} &= \left[ -CS_i + R(\Phi[D(p_i)]) - c(\Phi[D(p_i)]) \right. \\ &\quad \left. - C_i(D(p_i)) \right] P_i(V(CS_i)). \end{aligned} \tag{15}$$

It is clear, however that for any choice of  $CS_i$ , the right-hand side of (15) is maximized when  $D(p_i)$  maximizes the expression in the square bracket. But then  $D(p_i) = y_i^*$  defined by Eq. (2) as the input quantity that would yield production efficiency for bidder  $i$ .  $\square$

Once the bidding of a variable price equal to marginal cost at the optimal production level has been established as a dominant strategy, the first price auction reduces to a standard one-dimensional auction problem. As was shown by Myerson (1981) for the one-dimensional case, the first price auction, under several important restrictions, will result in equivalent allocations and payments as a second price auction, or an oral auction. Under the assumption of symmetric, independent beliefs amongst bidders and the risk neutrality of bidders, therefore, the first price auction would be selection efficient as we have defined it. Since these assumptions are somewhat tenuous, we now turn our attention to the second price auction, whose efficiency will not depend upon either of these restrictions.

**5. Second price auction**

We will now consider second price auctions in which the scoring rule is again a monotone function of the purchaser surplus while the fixed payment to the winning bid,  $\hat{F}$ , is determined, as described earlier, based on the winning variable price and the best losing score. Recall that  $\hat{F}_i$  is set so that  $S(\hat{F}_i, p_i) = S(F_j, p_j)$ , where  $\{F_j, p_j\}$  is the best losing bid. Based on Eq. (10) the fixed payment to winning bid  $i$  is thus

$$\begin{aligned} \hat{F}_i &= F_j - \int_{p_j}^{\infty} D(p) dp + \int_{p_i}^{\infty} D(p) dp \\ &= F_j - \int_{p_j}^{p_i} D(p) dp. \end{aligned} \tag{16}$$

Bidder  $i$  forms a subjective probability distribution over the score  $S_j$  corresponding to the best compet-

ing bid. Let  $G_i(s) = G_i\{S_j \leq s\}$  denote the subjective probability of bidder  $i$  that the score corresponding to the best competing bid does not exceed  $s$ . Substituting (16) and (14) into (5) and expressing the fixed payments in terms of the corresponding best losing score and the marginal transfer price yields the expected payoff <sup>5</sup> for bidder  $i$ :

$$\begin{aligned} &E\{\pi_i^{\text{II}}(S_i, p_i)\} \\ &= \int_{-\infty}^{S_i} \left[ -s + \int_{p_i}^{\infty} D(p) dp + p_i D(p_i) \right. \\ &\quad \left. - C_i(D(p_i)) \right] dG_i(s) \\ &= \int_{-\infty}^{S_i} \left[ -s + R(\Phi[D(p_i)]) - c(\Phi[D(p_i)]) \right. \\ &\quad \left. - C_i(D(p_i)) \right] dG_i(s). \end{aligned} \tag{17}$$

We will assume that the outside integral in (17) is a Stieltjes integral to accommodate possible mass point in the subjective probability distribution.

**Proposition 4.** *In the second price auction described above, where the scoring formula is given by (10) and the fixed payment to the winning bid is as in (16), it is a dominant strategy for a bidder to bid a variable price that maximizes total profit under its cost function and a fixed price which equals its producer's surplus, defined as  $D(p_i)p_i - C_i(D(p_i))$ , under that variable price.*

**Proof.** Clearly, selecting a strategy  $\{F_i, p_i\}$  is equivalent to choosing the corresponding pair  $\{S_i, p_i\}$ . Furthermore, regardless of the choice of  $S_i$ , the right-hand side of (17) is maximized pointwise when  $p_i$  is chosen so that  $D(p_i)$  maximizes the integrand in the square bracket. This is achieved at a price  $p_i^*$ , where  $D(p_i^*) = y_i^*$ , the efficient production level defined by (2). Substituting that quantity into (17)

<sup>5</sup> For simplicity we will exclude the possibility of ties although those can be easily handled by a straightforward generalization.

and denoting the total corporate profit (Eq. (1)) corresponding to that quantity as  $\Pi_i^*$  gives

$$\begin{aligned}
 E\{\pi_i^{\text{II}}(S_i, p_i^*)\} &= \int_{-\infty}^{S_i} [-s + \Pi_i^*] dG_i(s) \\
 &= \int_{-\infty}^{\Pi_i^*} [-s + \Pi_i^*] dG_i(s) \\
 &\quad + \int_{\Pi_i^*}^{S_i} [-s + \Pi_i^*] dG_i(s). \tag{18}
 \end{aligned}$$

Note that Eq. (18) is analogous to the expected profit equation in a standard Vickrey auction where  $-S_i$  represents the bid and  $-\Pi_i^*$  the private cost. The last integral in (18) is clearly non-positive since the differential is positive and either the integrand is negative or the upper limit is smaller than the lower limit. Thus,

$$E\{\pi_i^{\text{II}}(S_i, p_i^*)\} \leq \int_{-\infty}^{\Pi_i^*} [-s + \Pi_i^*] dG_i(s), \tag{19}$$

and the maximum expected payoff is achieved when  $S_i = \Pi_i^*$ , or equivalently using (1):

$$\begin{aligned}
 -F_i + \int_{p_i^*}^{\infty} D(p) dp &= R(\Phi[D(p_i^*)]) - c(\Phi[D(p_i^*)]) \\
 &\quad - C_i(D(p_i^*)).
 \end{aligned}$$

Consequently from (14),

$$F_i = -p_i^* D(p_i^*) + C_i(D(p_i^*)), \tag{20}$$

which is exactly the net cost of supply at the production efficient quantity.  $\square$

**Corollary 1.** *The outcome of the second price auction satisfies both production efficiency and selection efficiency criteria. Furthermore, the profit of the winning bid equals its total profit advantage over the best losing bid.*

**Proof.** From Proposition 4, the winning bidder will bid a production efficient variable price and a fixed price given by (20). The purchaser surplus corresponding to the winning bid therefore equals the corresponding total corporate profit. This implies

that the winning bid is the one that achieves the highest total profit at its production efficient marginal transfer price. The net payoff to the winner is obtained from (5), (16) and (20) as

$$\begin{aligned}
 \pi_i^{\text{II}}(F_i, p_i) &= \hat{F}_i + p_i^* D(p_i^*) - C_i(D(p_i^*)) \\
 &= F_j - \int_{p_j}^{p_i^*} D(p) dp + p_i^* D(p_i^*) - C_i(D(p_i^*)) \\
 &= -p_j^* D(p_j^*) + C_j(D(p_j^*)) - \int_{p_j}^{p_i^*} D(p) dp \\
 &\quad + p_i^* D(p_i^*) - C_i(D(p_i^*)) \\
 &= \Pi_i^* - \Pi_j^*,
 \end{aligned}$$

where  $j$  denotes the best losing bid.  $\square$

## 6. Conclusion

We have developed a foundation for modeling the simultaneous selection of a supplier of an intermediate product and the production levels for that product. In companies whose divisions operate as independent cost or profit centers, the auction process provides a tool for efficiently communicating information between divisions. They can be designed to provide incentives for the efficient revelation of information known only to individual suppliers. Auctions also establish common sense rules of procedure that can reduce the internal transaction costs and political strife that can arise through less formalized procedures.

We have developed two bidding mechanisms based on a scoring rule that induces potential producers to reveal truthfully their relevant cost information and achieves production and selection efficiency. The formulation of an internal demand function for the intermediate product serves to transmit information about the value of that product upstream to the suppliers. The mechanisms do not depend on any distributional assumptions about supplier ‘types’ and can be viewed as two-dimensional revelation mechanisms since the relevant private information could be characterized by a general two-dimensional distribution. In the case of the second price auction, both parameters are, in dominant strategies, truth-



fully revealed and the winning bidder retains the difference in total profit between itself and the second best supplier as information rent.

It is useful to compare these results to those that would arise from applying the most theoretically elegant centralized mechanism, the Groves mechanism (for a survey of implementation theory, see (Moore, 1992) and (Palfrey, 1992)). With the Groves mechanism, the purchasing division would ask the suppliers to provide their complete cost functions and use them to optimize company wide production. Those divisions that supply intermediate products would be rewarded according to some monotone function of the entire company's profit. Under the conventions of our model, such a mechanism could also result in production and selection efficiency and have the added advantage of being applicable to environments where the purchaser is not constrained, as he is in our model, to selecting a single supplier for the intermediate product.

Theorists have been trying to reconcile the apparent optimality of a highly centralized management structure and the decentralized corporate structure that is often observed in practice (for example, (Vaysman, 1993)). Much recent attention has focused on the real life constraints on information transfer as a key factor favoring decentralization (Melumad, Mookherjee and Reichelstein). In this paper, the efficiency of information transfer is manifested in both the signaling of purchaser preferences through a scoring rule and the description of a potentially complex cost function with only two parameters, a fixed and marginal transfer.

We have shown how the scoring of fixed and marginal transfer price bids communicates the values of the intermediate and final products of a corporation. A close examination of the bidding procedures used in industry today could reveal the nature of the preferences that are being communicated to suppliers. An empirical analysis of how internal bids are evaluated in comparison to external bids (according to our model, they should be treated differently) could serve as an indicator of how rationally companies have pursued decentralization and the exploitation of external markets for their intermediate products.

An interesting extension is to examine the incentives of the purchasing division if it too was a profit

center with private information. One possible approach would be to design a contingent contract for the purchasing division that induces it to reveal the true assembly costs. The internal demand function for  $y$  would then be adjusted to reflect the higher virtual cost (which would include the purchasing division's information rent) of producing the final product,  $x$ .

If the purchasing division's goal is to maximize purchaser surplus instead of the profit of the company as a whole, it is possible that it would want to induce marginal transfer price bids that are greater than true marginal cost in order to decrease the information rent collected by the supplying division. In a one-dimensional revelation problem, it was demonstrated by Riordan and Sappington (1987) that a mechanism which distorts marginal cost can perform better from the purchaser's point of view than one which achieves the organization's optimal solution. This gain is achieved by employing two information components (transfer and price) to reveal a one-dimensional type parameter. It is not clear, however, to what extent this gain is still possible when two dimensions of private information need to be revealed via a two-dimensional auction as in our model. In standard private information models, the information rent can be reduced without sacrificing selection efficiency since the low cost producer is the same for all production levels. In this model, a change in the scoring rule that reduces the information rent of winning bidders will also change the identity of who that winning bidder is. The optimal mechanism in such a circumstance remains an open question.

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